Guía Conceptual de Matemática.
Tema: Desarrollo de Fórmulas de Figuras Clásicas.
Montoya

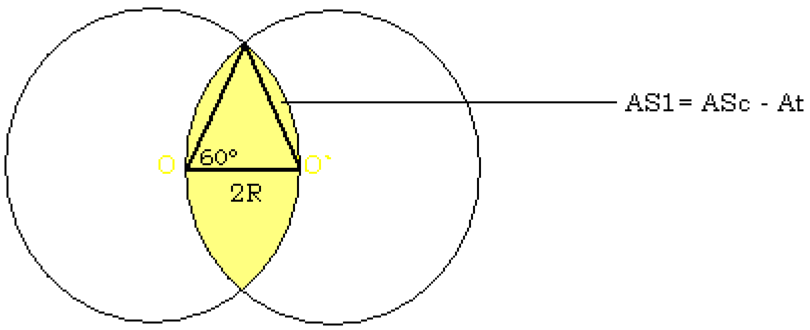
Conceptos previos

¡ESTO ES VERDADERAMENTE MARAVILLOSO!

¡Les invito a regocijarse espiritualmente con este tema!

Obs: Algunos nombres de las figuras han sido asignados arbitrariamente atendiendo a las características de la misma, otros sin embargo, conservan el nombre asignado oficialmente.

<u>2 Círculos</u>



Desarrollo:

$$AS1=\left(\frac{\alpha}{360^\circ}\pi R^2 - 1/4\, L^2\sqrt{3}\right) x\, 4 + 2\ 1/4\, R^2\sqrt{3}$$

$$AS1=\left(\frac{\cancel{60^\circ}}{\cancel{360^\circ}}\pi\, \cancel{4}R^2 - 1/\cancel{4}\,\cancel{4}R^2\sqrt{3}\right) x\, 4 + 2\ 1/\cancel{4}\,\cancel{4}R^2\sqrt{3}$$

$1/\cancel{6}$ → 3; $\cancel{4}$ → 2

$$AS1=\left(1/3\,\pi\, 2R^2 - R^2\sqrt{3}\right) x\, 4 + 2R^2\sqrt{3}$$

$$AS1= 8/3\,\pi R^2 - 4R^2\sqrt{3} + 2R^2\sqrt{3}$$

$$AS1= 8/3\,\pi R^2 - 2R^2\sqrt{3}$$

$$AS1= \frac{8\pi R^2 - 6R^2\sqrt{3}}{3}$$

$$AS1= 2/3\, R^2\left(4\pi - 3\sqrt{3}\right)$$

Ejemplos:

r = 3 r = 6

$AS1 = 2/3\ R^2\ (4\pi - 3\sqrt{3})$

$2/3\ 3^2\ (4\pi - 3\sqrt{3})$ $2/3\ 6^2\ (4\pi - 3\sqrt{3})$

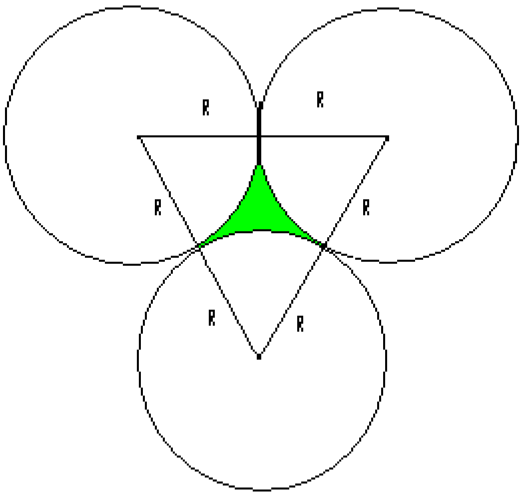
$2/3\ 9\ (4\pi - 3\sqrt{3})$ $2/3\ 36\ (4\pi - 3\sqrt{3})$

$6\ (4\pi - 3\sqrt{3})$ $24\ (4\pi - 3\sqrt{3})$

SUPERFICIE COMPRENDIDA POR UN TRIO DE CÌRCULOS CONGRUENTES TANGENTES EXTERIORMENTE

LA FIGURA SIGUIENTE SE FORMA CON TRES CIRCUNFERENCIAS CONGRUENTES (RADIOS IGUALES) TANGENTES EXTERIORMENTE.

3 Círculos



Desarrollo Analítico:
Calculamos la superficie del sector circular de ángulo central de 60ª.

$S1 = 60/360 \cdot \pi \cdot r^2$

$= 1/6\ \pi\ r^2$

Ahora calculamos el área del triángulo equilátero cuyo lado mide 2r-

$$S2 = \frac{4r^2\sqrt{3}}{4}$$

$= r^2\ \sqrt{3}$

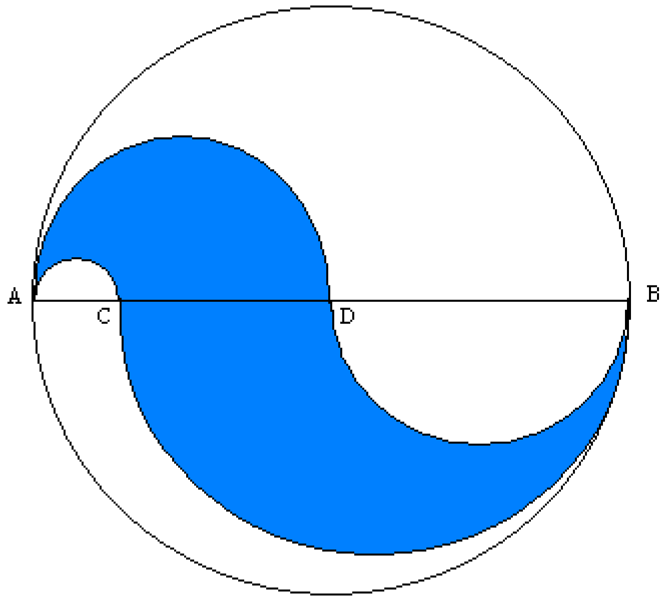
$AS = r^2\sqrt{3} - 3\,(1/6\,\pi\, r^2)$

$= r^2\sqrt{3} - ½\,\pi\, r^2$

$= r^2(\sqrt{3} - \frac{1}{2}\pi)$

$A_S = \frac{r^2}{2}(2\sqrt{3} - \pi)$

<u>Babosa 1</u>



PROPOSICION: Las superficies están en relación a los trazos AC; CD y DB, en que se divide el diámetro.

<u>Demostración:</u>

$A1 = ½\,\pi\left(\frac{a+b+c}{2}\right)^2 - ½\,\pi\left(\frac{b+c}{2}\right)^2 + ½\,\pi\,(a/2)^2$

$A2 = ½\,\pi\left(\frac{a+b}{2}\right)^2 - ½\,\pi\,(a/2)^2 + ½\,\pi\left(\frac{b+c}{2}\right)^2 - ½\,\pi\,(c/2)^2$

$A3 = ½\,\pi\left(\frac{a+b+c}{2}\right)^2 - ½\,\pi\left(\frac{a+b}{2}\right)^2 + ½\,\pi$

$A1 = 1/8\,\pi\{(a+(b+c))^2 - (b+c)^2 + (a)^2\}$

$A2 = 1/8\,\pi\{(a+b)^2 - a^2 + (b+c)^2 - c^2\}$

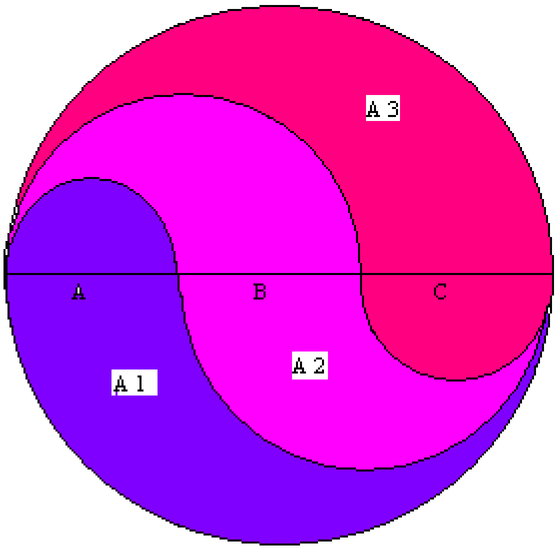
$A3 = 1/8\,\pi\{(a+b) + c)^2 - (a+b)^2 + (c)^2\}$

$A1 = 1/8\ \pi\{2a^2+2ab+2ac\}$
$A2 = 1/8\ \pi\{2ab+2ab^2+2bc\}$
$A3 = 1/8\ \pi\{2ac+2bc+2\ c^2\}$

$A1 = 1/8\ \pi\ 2\ a\ (a+b+c)$
$A2 = 1/8\ \pi\ 2\ b\ (a+b+c)$
$A3 = 1/8\ \pi\ 2\ c\ (a+b+c)$

S1: S2: S3 = a: b: c

LA FIGURA SE CONSTRUYE DIVIDIENDO EL DIÀMETRO DE LA CIRCUNFERENCIA EN TRES PARTES CON LO QUE SE CONSTRUYEN LAS CURVAS CERRADAS QUE SE MUESTRAN.



Relación de las áreas A, B, C:

$$A1 = ½\ \boldsymbol{\pi} \left(\frac{a+b+c}{2}\right)^2 - ½\ \boldsymbol{\pi} \left(\frac{b+c}{2}\right)^2 + ½\ \boldsymbol{\pi} \left(\frac{a}{2}\right)^2$$

$$= ½\ \boldsymbol{\pi} \left[\left(\frac{a+b+c}{2}\right)^2 - \left(\frac{b+c}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]$$

$$= 1/8\ \boldsymbol{\pi}\ (a+b+c)^2 - (b+c)^2 + (a)^2$$

$= 1/8\, \boldsymbol{\pi}\, [a^{2} + 2a\,(a+b) + (b+c)^2 - (b+c)^2 + a^2$

$= 1/8\, \boldsymbol{\pi}\, [2a^2 + 2ab + 2ac\,]$

$= ¼\, \boldsymbol{\pi}\;\; a\,(a+b+c)$

$A\,2 = ½\, \boldsymbol{\pi}\, \left(\frac{a+b}{2}\right)^2 – ½\, \boldsymbol{\pi}\, \left(\frac{a}{2}\right)^2 + ½\, \boldsymbol{\pi}\, \left(\frac{b+c}{2}\right)^2 – ½\, \boldsymbol{\pi}\, \left(\frac{c}{2}\right)^2$

$= 1/8\, \boldsymbol{\pi}\, [(a+b)^2 – a^2 + (b+c)^2 – c^2]$

$= 1/8\, \boldsymbol{\pi}\, [a^2 + 2ab + b^2 + a^2 + b^2 + 2bc + c^2 – c^2]$

$= 1/8\, \boldsymbol{\pi}\;\; [2ab + 2b^2 + 2bc]$

$= 1/8\, \boldsymbol{\pi} \cdot 2b\,(a+b+c)$

$= ¼\;\; \boldsymbol{\pi}\, b(a+b+c)$

$A\,3 = ½\;\; \boldsymbol{\pi}\, \left(\frac{a+b+c}{2}\right)^2 – ½\, \boldsymbol{\pi}\, \left(\frac{a+b}{2}\right)^2 + ½\, \boldsymbol{\pi}\, \left(\frac{c}{2}\right)^2$

$= 1/8\, \boldsymbol{\pi}\, [(a+b+c) – (a+b) + c]$

$= 1/8\, \boldsymbol{\pi}\, [(a+b)^2 + 2c\,(a+b) + c^2 – (a+b)^2 + c^2$

$= 1/8\, \boldsymbol{\pi}\, [2ac + 2bc + 2c^2]$

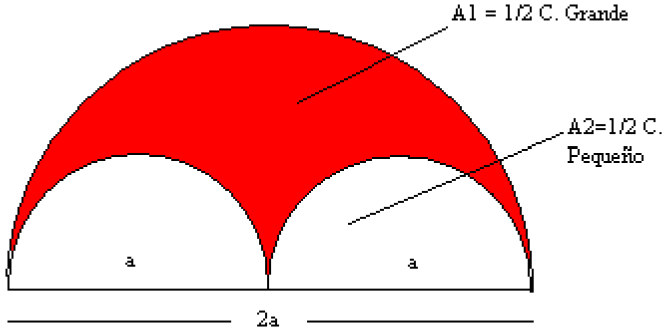
$= 1/8\, \boldsymbol{\pi} \cdot 2c\, [a+b+c]$

$= ¼\;\; \boldsymbol{\pi}\, c\, [a+b+c]$

S1: S2: S3 = a: b: c

ES DECIR: LAS SUPERFICIES ENCERRADAS POR LAS CURVAS SON PROPORCIONALES A LAS MEDIDAS DE LOS TRAZOS EN QUE SE DIVIDE EL DIÀMETRO.

Bisemicircular



Desarrollo:

A1: ½ $(4\ a)^2\ \pi$ → ½ 16 $a^2\ \pi$ → 8 $a^2\ \pi$
A2: ½ $(2\ a)^2\ \pi$ → ½ 4 $a^2\ \pi$ → 2 $a^2\ \pi$

A3 (Figura Roja): A1 – 2 A2
$8\ a^2\ \pi - 2\ (2\ a^2\ \pi)$
$8\ a^2\ \pi - 4\ a^2\ \pi$
$4\ a^2\ \pi$

Ejemplos:

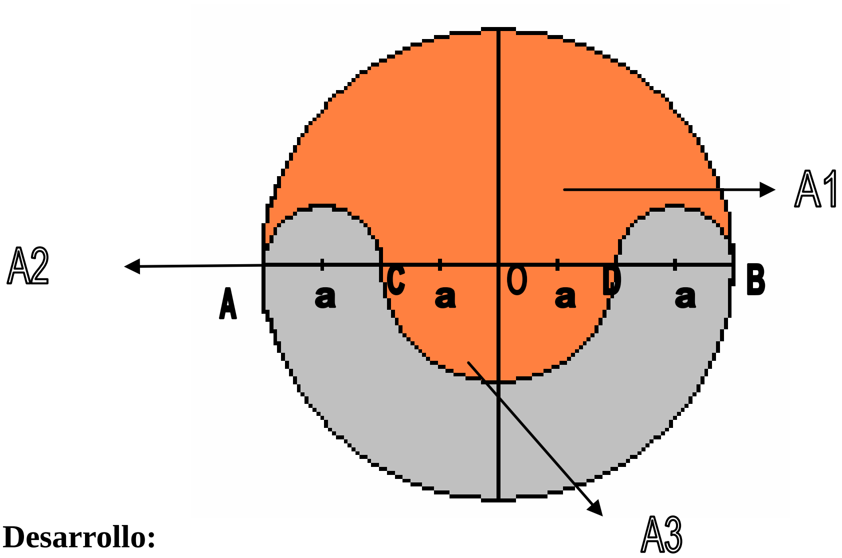
1.- Si a = 5, entonces:
Radio A1 = 20 A1: ½ $20^2\ \pi$ → $200\ \pi$
Radio A2 = 10 A2: ½ $10^2\ \pi$ → $50\ \pi$

A3: A1 – 2 A2
$200\ \pi - 2(50\ \pi)$
$100\ \pi$

2.- Si a = 12, entonces:
Radio A1 = 48 A1: ½ $48^2\ \pi$ → $1152\ \pi$
Radio A2 = 24 A2: ½ $24^2\ \pi$ → $288\ \pi$

A3: A1 – 2 A2
$1152\ \pi - 2(288\ \pi)$
$576\ \pi$

Carambola



Desarrollo:

A1: $\frac{1}{2} (4\,a)^2\,\pi \rightarrow \quad \frac{1}{2}\,16\,a^2\,\pi \rightarrow \quad 8\,\pi\,a^2$

A2: $\frac{1}{2} (2\,a)^2\,\pi \rightarrow \quad \frac{1}{2}\,4\,a^2\,\pi \rightarrow \quad 2\,\pi\,a^2$

A3: $\frac{1}{2} (a)^2\,\pi \rightarrow \quad \frac{1}{2}\,a^2\,\pi \rightarrow \quad \frac{1}{2}\,a^2\,\pi$

Área Naranja: A1 + A2 - 2 A3

$8\,\pi\,a^2 + 2\,\pi\,a^2 - 2\,(\frac{1}{2}\,\pi\,a^2)$

$9\,\pi\,a^2$

* El área amarilla está en relación al área roja en razón 9:7

Área Gris: A1 + 2 A3 – A2

$8\,\pi\,a^2 + 2\,(\frac{1}{2}\,\pi\,a^2) - 2\,\pi\,a^2$

$7\,\pi\,a^2$

Ejemplos:

1.- Si a = 4, entonces:

Radio A1 = 16 — A1: $\frac{1}{2}\,16^2\,\pi \rightarrow 128\,\pi$

Radio A2 = 8 — A2: $\frac{1}{2}\,(8)^2\,\pi \rightarrow 32\,\pi$

Radio A3 = 4 — A3: $\frac{1}{2}\,(4)^2\,\pi \rightarrow 8\,\pi$

Por lo tanto:

Área Naranja: A1 + A2 - 2 A3

$128\,\pi + 32\,\pi - 2\,(8\,\pi)$

$112\,\pi$

Área Gris: A1 + 2 A3 – A2

$128\,\pi + 2\,(8\,\pi) - 32\,\pi$

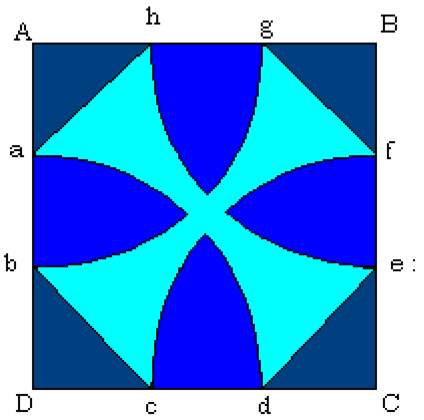
$144\,\pi$

* $\frac{112}{144} = \frac{7}{9}$

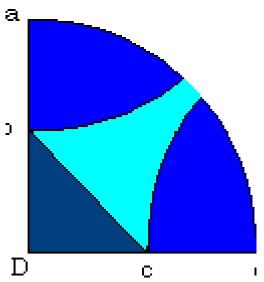
CRUZ DE MALTA.

SE CONSTRUYE COMO SE INDICA:
1.- SE TRAZAN LAS DIAGONALES DEL CUADRADO.
2.- HACIENDO CENTRO EN CADA VÈRTICE, SE TRAZAN LOS CUARTOS DE CÌRCULO DE RADIO EQUIVALENTE A LA MITAD DE LA DIAGONAL DEL CUADRADO
3.- SE UNEN DOS A DOS LOS PUNTOS DE INTERSECCIÒN DE ESTOS CUARTOS DE CIRCULOS CON CADA UNO DE LOS LADOS CONTIGUOS DEL CUADRADO.

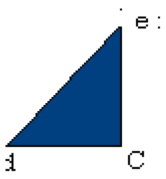
Cruz de Malta



DEDUCCIÒN DE LA MÈTRICA DE LA CRUZ DE MALTA



$= \frac{1}{4} \pi \times (3a)^2$



$= a^2/2$

Cruz de malta = cuadrado – 2 x sector circular + 4 x triangulo

$\{(4\ a)^2 – ½\ \pi\ (3\ a)^2 + 2\ a^2\}2$

{16 a2 – ½ π 9 a2 + 2 a2}2

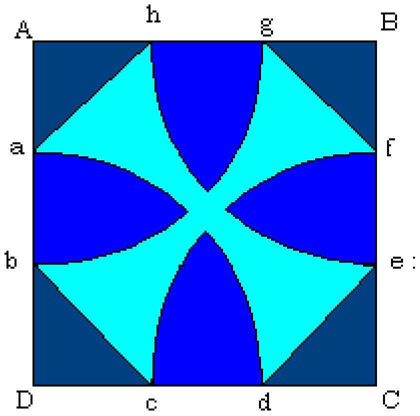
{32 a2/2 – π 9 a2/2 + 4 a2/2}2

$\frac{\{36\ a2 - 9\ \pi\ a2\}2}{2}$

{9 a2/2(4- π)}

Resultado = 9 a2 (4 – π)

Ejemplos:



a = 2

9 a2 (4 – π)

9 · 4 (4 – π)

36 (4 – π)

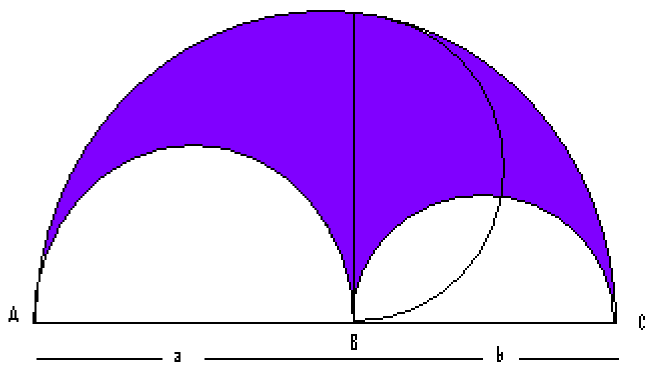
a = 5

9 a2 (4 – π)

9· 10 (4 – π)

90 (4 – π)

<u>El arbelus de Arquímedes</u>



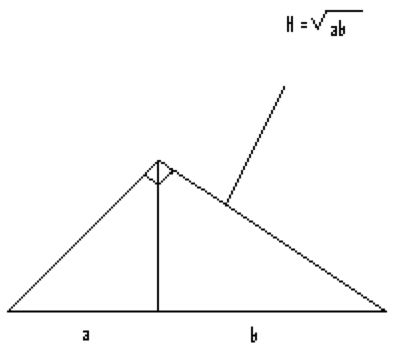
Demostración:

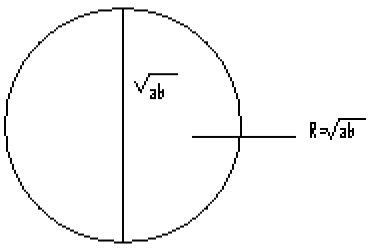
$$As = \frac{1}{2}\,\pi \left(\frac{a+b}{2}\right)^2 - \frac{1}{2}\,\pi \left(\frac{a}{2}\right)^2 - \frac{1}{2}\,\pi \left(\frac{b}{2}\right)^2$$

$$As = 1/8\,\pi\,(a+b)^2 - 1/8\,\pi\,a^2 - 1/8\,\pi\,b^2$$

$$As = 1/8\,\pi\,2ab$$

$$As = \frac{1}{4}\,\pi\,ab$$



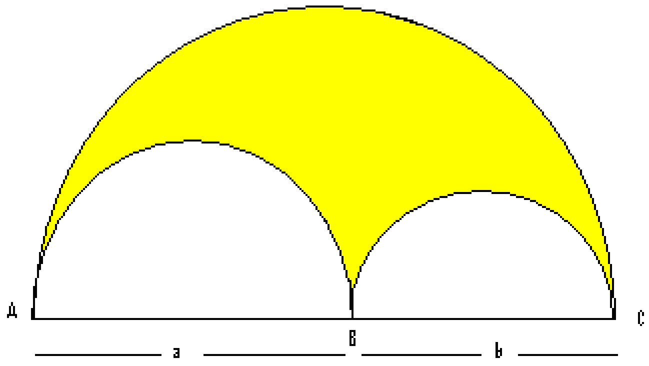


$A\otimes = \pi\,(1/2\ \sqrt{ab})^2$

$= \pi \cdot \frac{ab}{4}$

$= \frac{1}{4}\,ab\,\pi$ -> El área del semi círculo es igual al área sombreada en el arbelus.

Ejemplos:



a = 20
b = 12

As = ¼ **π** ab

As = ¼ **π** 20·12

As = ¼ **π** 240

As = 60 **π**

a = 40
b = 15

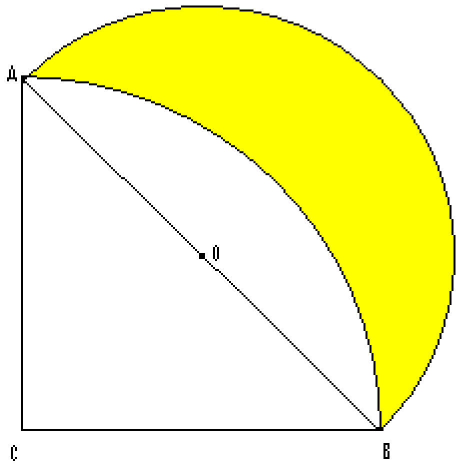
As = ¼ **π** ab

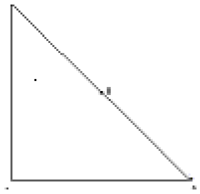
As = ¼ **π** 40·15

As = ¼ **π** 600

As = 150 **π**

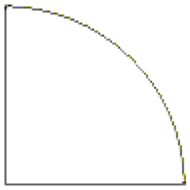
<u>Lúnula</u>





A 1 = ½ · 2 r · 2r

= 2 r^2

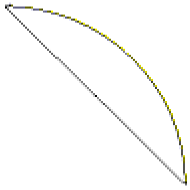


A 2 = ¼ $\boldsymbol{\pi}$ $(2\ r)^2$

= ¼ $\boldsymbol{\pi}$ 4 r^2

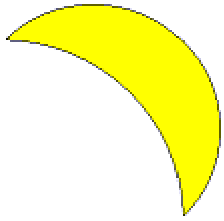
= $\boldsymbol{\pi}$ r^2

A1 – A2
$\boldsymbol{\pi}\ r^2 - 2\ r^2$



As = ½ $\boldsymbol{\pi}$ $(r^2\sqrt{2}^2)$

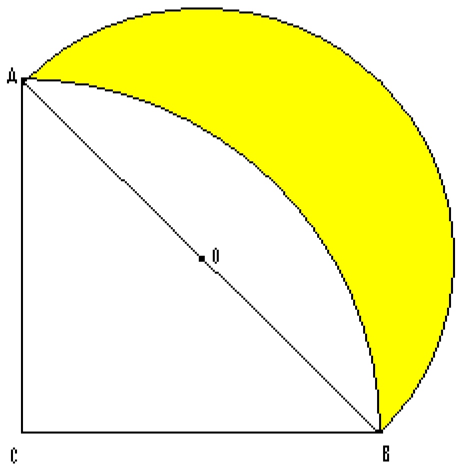
$= ½ \boldsymbol{\pi} \cdot r^2 \cdot 2$
$= \boldsymbol{\pi} r^2$



$As = \boldsymbol{\pi} r^2 – (\boldsymbol{\pi} r^2 - 2 r^2)$

$Al = 2 r^2$

Ejemplos:



1.- Si r = 5,

A5 = ½ r^2

= ½ 25

= 12.5

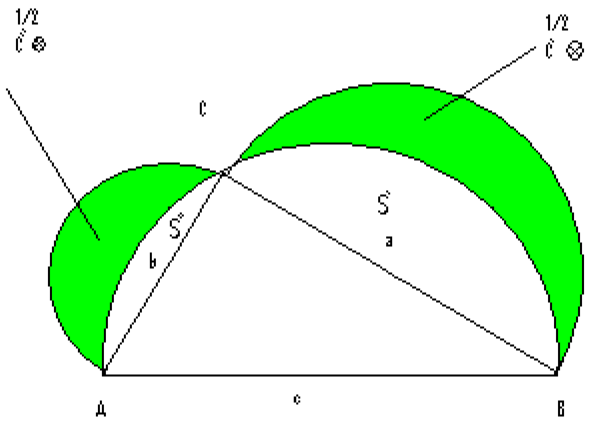
2.- Si r = 20,

A5 = ½ r^2

= ½ 400

= 200

<u>**La Creciente de Hipócrates**</u>



Demostración:

$*A\ \frac{1}{2}\ C\otimes` = \frac{1}{2}\ \pi \left(\frac{b}{2}\right)^2 = 1/8\ \pi\ b^2$

$A\ \frac{1}{2}\ C\otimes`` = \frac{1}{2}\ \pi \left(\frac{a}{2}\right)^2 = 1/8\ \pi\ a^2$

$S` + S`` = 1/8\ \pi\ c^2 - \frac{1}{2}\ ab$

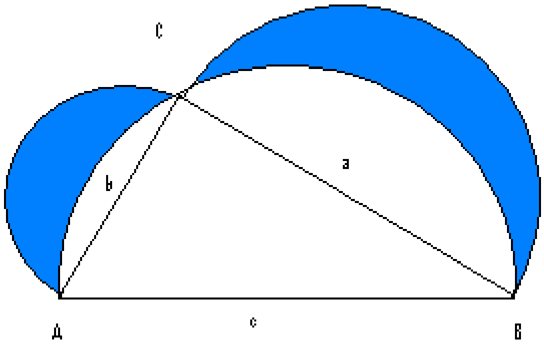
$*As = 1/8\ \pi\ b^2 + 1/8\ \pi\ a^2 - (1/8\ \pi\ c^2 - \frac{1}{2}\ ab)$

$As = 1/8\ \pi\ b^2 + 1/8\ \pi\ a^2 - 1/8\ \pi\ c^2 + \frac{1}{2}\ ab$

$= 1/8\ \pi\ (a^2+b^2) - 1/8\ \pi\ c^2 + \frac{1}{2}\ ab$

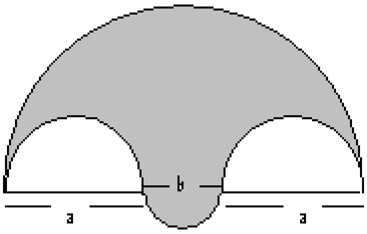
$As = \frac{1}{2}\ ab$ → El área de la Creciente de Hipócrates corresponde al área del triángulo

Ejemplos:



a = 10 b = 8 c = 12	a = 4 b = 2 c = 6
As = ½ ab	As = ½ ab
As = ½ · 10·8	As = ½ · 2·4
As = ½ · 80	As = ½ · 8
As = 40	As = 4

<u>**Salinum de Arquímedes**</u>



Demostración:

$As = \frac{1}{2}\ \pi \left(\frac{2\ a + b}{2}\right)^2 - \frac{1}{2}\ \pi\ (1/2)^2 - \frac{1}{2}\ \pi\ (a/2)^2 + \frac{1}{2}\ \pi\ (b/2)^2$

$As = 1/8\ \pi\ (2\ a + b)^2 - 1/8\ \pi\ a^2 - 1/8\ \pi\ a^2 + 1/8\ \pi\ b^2$

$As = 1/8\ \pi\ [(2\ a + b)^2 - a^2 - a^2 + b^2]$

$As = 1/8\ \pi\ [4\ a^2 + 4\ ab + b^2 - a^2 - a^2 + b^2]$

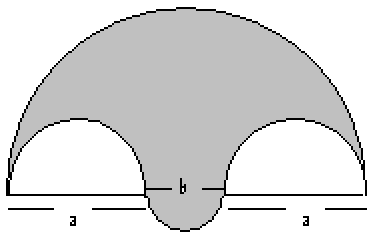
$As = 1/8\ \pi\ [2\ a^2 + 4\ ab + 2\ b^2]$

$As = 1/8\ \pi \cdot 2\ [a^2 + 2\ ab + b^2]$

$As = \frac{1}{4}\ \pi\ (a+b)^2$

$As = \pi\ \frac{(a+b)^2}{2}$ → Fórmula Final.

Ejemplos:



1. a = 4 b = 2

   a = 6 b = 2

2. $As = \pi \frac{(a+b)^2}{2}$

   $As = \pi \frac{(a+b)^2}{2}$

   $As = \pi \frac{(4+2)^2}{2}$

   $As = \pi \frac{(6+2)^2}{2}$

   $As = \pi \frac{(16+4)}{2}$

   $As = \pi \frac{(36+4)}{2}$

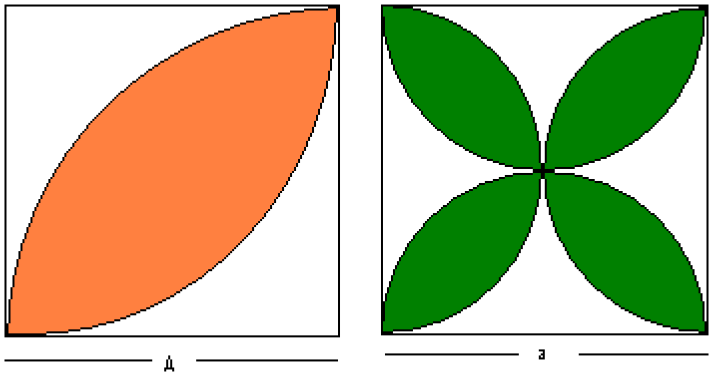
   $As = \pi \frac{20}{2}$

   $As = \pi \frac{40}{2}$

   $As = 10\pi$

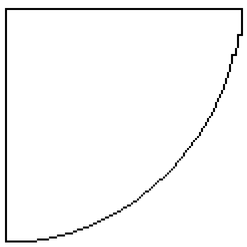
   $As = 20\pi$

<u>Casi /Trébol</u>



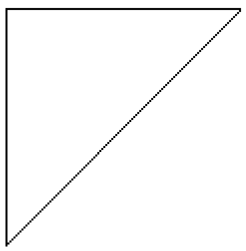
Desarrollo:

Area 1:



$$\frac{1}{4}\pi a^2$$

Area 2:



$$\frac{b \times h}{2} = \frac{a \times a}{2} = \frac{a^2}{2}$$

Area sector
achurado: área 1 - área 2.

$$Asa = 1/4\,\pi a^2 - \frac{a^2}{2}$$

$$Asa = \frac{2\pi a^2 - a^2}{4}$$

$$Asa = 2\left(\frac{a^2}{4}(\pi - 2)\right)$$

$$Asa = \not{2} \times \frac{a^2}{\not{4}\;2}(\pi - 2)$$

$$Asa = \frac{a^2}{2}(\pi - 2)$$

Ejemplos:

Figura 1

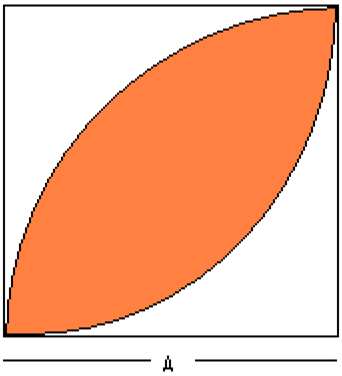


Figura 2

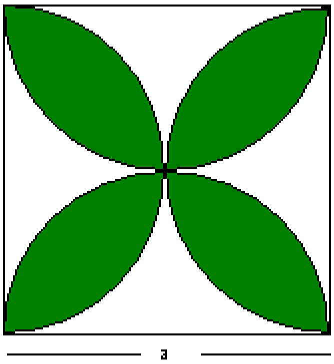


Figura 1:

a = 4

$$Asa = \frac{a^2}{2}(\pi - 2)$$

$$= \frac{4^2}{2}(\pi - 2)$$

$$= 16/2\,(\pi - 2)$$

$$= 8(\pi - 2)$$

a = 6

$$Asa = \frac{a^2}{2}(\pi - 2)$$

$$Asa = \frac{6^2}{2}(\pi - 2)$$

$$= 36/2\,(\pi - 2)$$

$$= 18\,(\pi - 2)$$

Figura 2:

a = 8

$$= \frac{8^2}{2}(\pi - 2)$$

$$= 64/2\,(\pi - 2)$$

$$= 32\,(\pi - 2)$$

a = 10

$$= \frac{10^2}{2}(\pi - 2)$$

$$= 100/2(\pi - 2)$$

$$= 50(\pi - 2)$$